Therefore, the deviation from the stoichiometric composition to either side results in a reduction in the thermal conductivity of boron carbide. Two mechanisms can be proposed for the change in thermal conductivity. First, part of the excess boron or carbon atoms form a defect in the crystalline boron carbide structure which results in additional phonon scattering. The steeper slope of the right-hand branch in Fig. 2 is possibly associated with the fact that the large-scale boron atoms produce greater distortions in the structure than do the carbon atoms. Secondly, the excess boron or carbon atoms can exert an influence on the formation of the boron carbide microstructure and, particularly, on the degree of thermal contact between its grains, on which the magnitude of the thermal conductivity depends.

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DETERMINATION OF THE THERMAL DIFFUSIVITY OF MATERIALS BY MEASURING TEMPERATURES AT THE STAGE OF IRREGULAR OPERATION

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Explicit relations are obtained for the thermal diffusivity of materials on the basis of measurements of the temperatures at the stage of irregular operation in specimens of different geometrical shape (a plate, a cylinder, and a sphere).

The main aspect of existing methods of determining the thermal diffusivity of materials by measuring the temperatures at the stage of irregular operation is the solution of the direct problem of thermal conduction with boundary conditions obtained by experiment, and on the basis of this solution to obtain information on the required parameters. Since, as a rule, the solution of the problem is given in the form of an infinite series of transcendental functions, the thermal diffusivity cannot be expressed explicitly in terms of experimentally measured values of the temperature. However, it turns out to be possible to express the thermal diffusivity explicitly in terms of the measured values of temperature without solving the direct problem by using the integral Laplace transform. The use of the integral Laplace transform enables one to write the general solution of the one-dimensional thermal conduction equation

$$r^{1-k}\frac{\partial}{\partial r}\left(r^{k-1}\frac{\partial T}{\partial r}\right)=\frac{1}{a}\frac{\partial T}{\partial \tau} \quad (1)$$

in transform space in the following form:

$$T(r,\ s)=A\varphi_1(r,\ s)+B\varphi_2(r,\ s), \quad (2)$$

where k = 1, 2, 3 for a plane, cylindrical, and spherical field, respectively, while $\varphi_1(r,\ s)$ and $\varphi_2(r,\ s)$ are expressed in terms of hyperbolic Bessel functions [1]. The constants A and

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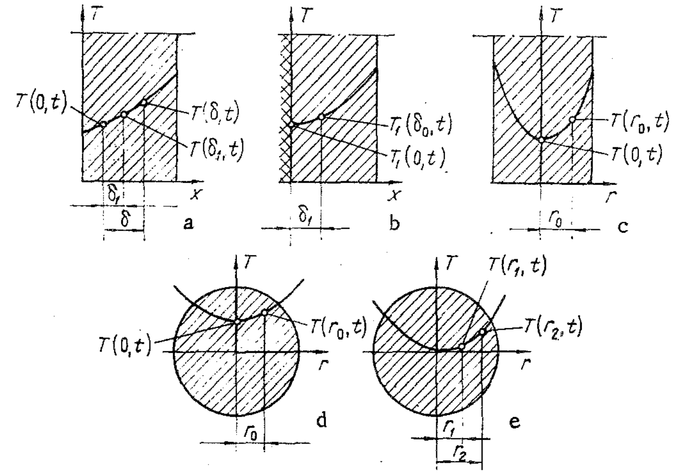

Fig. 1. Scheme showing the points where the temperature is measured in the specimens: a, b) in the plate; c) in the cylinder; d, e) in the sphere.

B are found from the boundary conditions. For an unbounded plane wall one can obtain from Eq. (2) the connection in transform space between the temperatures at three arbitrary points along the thickness (Fig. 1):

$$T(\delta_1,\ s) = T(0,\ s)\,\frac{\operatorname{sh}\sqrt{\frac{s}{a}}\,(\delta-\delta_1)}{\operatorname{sh}\sqrt{\frac{s}{a}}\,\delta} + T(\delta,\ s)\,\frac{\operatorname{sh}\sqrt{\frac{s}{a}}\,\delta_1}{\operatorname{sh}\sqrt{\frac{s}{a}}\,\delta}\,. \tag{3}$$

Obviously, when $\delta = 2\delta_1$, we have

$$\frac{T(0,\ s)+T(\delta,\ s)}{2T(\delta/2,\ s)} = \operatorname{ch}\sqrt{\frac{s}{a}}\,\frac{\delta}{2}\,. \tag{4}$$

We will put

$$\frac{T(0,\ s)+T(\delta,\ s)}{2} = T(s).$$

We differentiate the left and right sides of Eq. (4) with respect to s:

$$\left[\frac{T(s)}{T(\delta/2,\ s)}\right]' = \frac{\delta}{4\sqrt{sa}}\operatorname{sh}\sqrt{\frac{s}{a}}\,\frac{\delta}{2}\,. \tag{5}$$

Multiplying Eq. (5) by $\sqrt{s}$ and then differentiating it with respect to s, we obtain the expression

$$\left\{\left[\frac{T(s)}{T(\delta/2,\ s)}\right]'\sqrt{s}\right\}' = \frac{\delta^2}{16a\sqrt{s}}\operatorname{ch}\sqrt{\frac{s}{a}}\,\frac{\delta}{2}\,. \tag{6}$$

After carrying out the differentiation of the left side of Eq. (6), multiplying the right and left sides of Eq. (6) by $\sqrt{s}$ and $T^3(\delta/2,\ s)$, and assuming that $T(s) = T(\delta/2,\ s)\cosh\sqrt{(s/a)}(\delta/2)$, we obtain

$$\Big[T''(s)\,T^2\left(\frac{\delta}{2},\ s\right) - 2T'(s)\,T'\left(\frac{\delta}{2},\ s\right)T\left(\frac{\delta}{2},\ s\right) + 2T'^2\left(\frac{\delta}{2},\ s\right)\times$$

$$\times T(s) - T''\left(\frac{\delta}{2},\ s\right)T(s)\,T\left(\frac{\delta}{2},\ s\right)\Big]\,s + \frac{1}{2}\Big[T'(s)\,T^2\left(\frac{\delta}{2},\ s\right) -$$

$$- T'\left(\frac{\delta}{2},\ s\right)T(s)\,T\left(\frac{\delta}{2},\ s\right)\Big] = \frac{\delta^2}{16a}\,T(s)\,T^2\left(\frac{\delta}{2},\ s\right). \tag{7}$$

Transferring into the space of the originals using the well-known conversion formulas [2], we can express the thermal diffusivity explicitly in terms of the known values of the temperatures at three points of the plate connected by the relation

$$a = \frac{\delta_1^2}{4}\,\frac{\varphi[T(t),\ T(\delta/2,\ t)]}{\psi[T(t),\ T(\delta/2,\ t)]}\,. \tag{8}$$

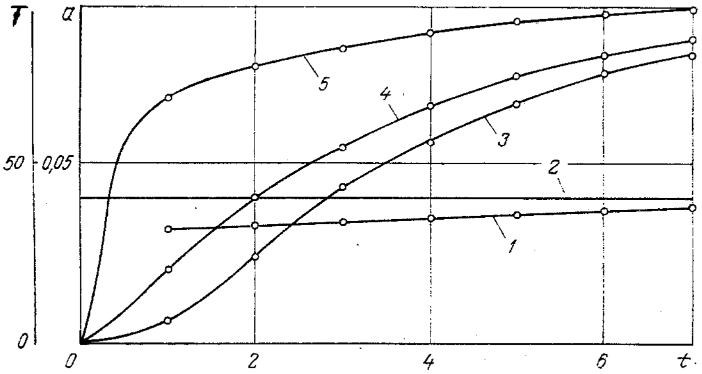

Fig. 2. Results of calculations of the thermal diffusivity: 1) calculated values of a, m^2/h; 2) assumed values of $a = 0.04\ m^2/h$; 3, 4, 5) temperatures at three points of the specimen, °C.

The functions φ and ψ are the following integral combinations:

$$\varphi(t) = \int_0^t T(\delta/2,\ t-\tau) \int_0^\tau T(\theta)\, T(\delta/2,\ \tau-\theta)\, d\theta d\tau, \tag{9}$$

$$\begin{aligned}\psi(t) = \int_0^t T(\delta/2,\ t-\tau) \int_0^\tau \Big\{(\tau-\theta)^2 \Big[T(\tau-\theta)\, \frac{dT(\delta/2,\ \theta)}{d\theta} - \\ - T(\delta/2,\ \tau-\theta)\, \frac{dT(\theta)}{d\theta}\Big] - \theta f\, [T(\theta)\, T(\delta/2,\ \tau-\theta) - T(\delta/2,\ \theta)\, T(\tau-\theta)]\Big\}\, d\theta d\tau \\ + 2\int_0^t (t-\tau)\, T(\delta/2,\ t-\tau) \int_0^\tau (\tau-\theta)\, \Big[T(\delta/2,\ \tau-\theta)\, \frac{dT(\theta)}{d\theta} - T(\tau-\theta)\, \frac{dT(\delta/2,\ \theta)}{d\theta}\Big]\, d\theta d\tau,\quad f = 1/2.\end{aligned} \tag{10}$$

Relations (8)-(10) also hold when the temperatures on the insulated side $T_1(0,\ t)$ and at a point at a distance δ_0 from it $T_1(\delta_0,\ t)$ are known. In this case it is sufficient to put $T(t) = T_1(\delta_0,\ t)$, $T(\delta/2,\ t) = T_1(0,\ t)$. The latter condition holds, since in Eq. (4) because of symmetry $T(0,\ t) = T(\delta,\ t)$, $\delta = 2\delta_0$. Calculations similar to those given above for the case of a solid cylinder for known temperatures on the axis $T(0,\ t)$ and at a distance r_0 from it $T(r_0,\ t)$ and also for a solid sphere, if the points at which measurements are made are connected by the relation $2r_1 = r_2$, lead to relations of the form (8)-(10). In this case for a solid cylinder $f = 1$, $T(t) = T(r_0,\ t)$, $T(\delta/2,\ t) = T(0,\ t)$, $\delta_1 = r_0$; for a solid sphere $f = 1/2$, $T(t) = T(r_2,\ t)$, $T(\delta/2,\ t) = T(r_1,\ t)$, $\delta_1 = r_1$. If in the case of a solid sphere the temperature at the center and at a distance r_0 from it are known, then $f = 3/2$, $T(t) = T(r_0,\ t)$, $T(\delta/2,\ t) = T(0,\ t)$, $\delta_1 = r_0$.

Since when making the calculations, to determine the thermal diffusivity from measurements of nonstationary temperatures in specimens of different configuration there is only a difference in the value of the coefficient f, for the cases considered it is possible to use a single calculation algorithm realized by means of a comparatively simple Algol program on a computer. It should be noted that for the case of a thermally insulated plane wall and a solid sphere, the range in which the points at which the temperature is measured are situated, for which explicit expressions are obtained for determining the thermal diffusivity, can be extended. The comparatively simple relations for the hyperbolic cosines and sines of multiple arguments enable one to obtain in transform space relations not containing transcendental functions and connecting the measured temperatures. The relation for determining the thermal diffusivity has a fairly simple form if a model of a semibounded body is used in the experiment. In this case, if the values of the temperature at distances x_1 and x_2 from the heated surface are known, the theoretical relations take the form

$$a = \frac{(x_2-x_1)^2}{4} \left\{\frac{\varphi[T(x_1,\ t)\, T(x_2,\ t)]}{\psi[T(x_1,\ t)\, T(x_2,\ t)]}\right\}^2, \tag{11}$$

$$\varphi(t) = \frac{1}{\sqrt{\pi}} \int_0^t T(x_2,\ t-\tau) \int_0^\tau \frac{T(x_1,\ \theta)}{\sqrt{(\tau-\theta)}}\, d\theta d\tau, \tag{12}$$

$$\psi(t)=\int_0^t \tau[T(x_2, \tau)\,T(x_1, t-\tau)-T(x_1, \tau)\,T(x_2, t-\tau)]\,d\tau. \tag{13}$$

Equations (11)-(13) are obtained by solving the thermal conduction equation in transforms:

$$T(x, s)=A\exp\left(-\sqrt{\frac{s}{a}}\,x\right). \tag{14}$$

The value of the coefficient A is found in terms of the known temperature at the point x_1 or x_2.

Hence, the relations obtained enable one to find the thermal diffusivity of materials from measurements of the nonstationary temperatures in specimens of different configuration. Unlike existing methods explicit equations are obtained for the required parameters in terms of the value of the temperatures, which eliminates the inconvenience involved in constructing and using tables or diagrams to determine these coefficients by solving the direct problem in the space of the originals.

To estimate the efficiency of the method we calculated the thermal diffusivity from the temperatures of an unbounded flat plate known from preliminary calculations with a known thermal diffusivity a = 0.04 m^2/h. The algorithm of the numerical method of calculation was realized using an Algol program on the BÉSM-4 computer.

The results obtained are shown in Fig. 2. As the known "experimental" values of the temperature we took the temperatures at three points 4 mm apart from one another. As is seen from the graphs, the values of the calculated thermal diffusivities are fairly close to those taken when solving the direct problem of the heating of the plate. The differences which occur are obviously due to errors in the numerical method of integration.

NOTATION

T, temperature of the body; x, r, coordinates; δ, δ_1, δ_0, distances between the points at which the temperature is measured in a flat unbounded plate; r_0, r_1, r_2, radii of the points at which the temperature is measured in specimens of cylindrical and spherical shape; a, thermal diffusivity; t, τ, θ, times.

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